

# Dynamic optimization: Continuous-time DSGE

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**Course objective.** This course provides a toolbox for solving continuous-time dynamic optimization problems in (stochastic) economic models. In particular, we briefly review optimal control theory and dynamic programming. We then thoroughly study models in continuous time under uncertainty. The optimization problems are illustrated by various examples of continuous-time dynamic stochastic general equilibrium (DSGE) models.

## Course outline.

### Part I: Basic mathematical tools

- (i) Control theory (maximum principle, Euler equation, transversality condition)
- (ii) Dynamic programming (Bellman equation, envelope theorem, multiple variables)
- (iii) An example: Lucas' model of endogenous growth

### Part II: Stochastic models in continuous time

- (i) Stochastic differential equations and rules for differentials (Itô's formula)
- (ii) An example: Merton's model of growth under uncertainty
- (iii) Stochastic dynamic control problems (Bellman equation)
- (iv) An example: Continuous-time RBC (under Gaussian and/or Poisson uncertainty)
- (v) An example: The matching approach to unemployment
- (vi) An example: Wälde's model of endogenous growth cycles

**Prerequisites.** Topics in integration, differential equations, qualitative theory, basic concepts of probability theory (Posch 2010, chap. 1.1-1.3, 1.7, 30 pages).

**Reading list.** The main references are Sydsæter et al. (2008, chap. 4-12, 290 pages) and Chang (2004, chap. 4, 50 pages). Reflecting the lecture notes (40 pages) is recommended. Various articles will be suggested as complementary material during the course.

## References

- Chang, Fwu-Ranq, *Stochastic optimization in continuous time*, Cambridge Univ. Press, 2004.
- Lucas, Robert E. Jr., “On the Mechanics of Economic Development,” *J. Monet. Econ.*, 1988, *22*, 3–42.
- Merton, Robert C., “An asymptotic theory of growth under uncertainty,” *Rev. Econ. Stud.*, 1975, *42* (3), 375–393.
- Posch, Olaf, *Advanced Macroeconomics*, <http://aula.au.dk/courses/2951/>: Lecture Notes, Aarhus University, 2010.
- Sydsæter, Knut, Peter Hammond, Atle Seierstad, and Arne Strøm, *Further Mathematics for Economic Analysis*, Prentice Hall, 2008.
- Wälde, Klaus, “Endogenous growth cycles,” *Int. Econ. Rev.*, 2005, *46* (3), 867–894.