

# Dynamic Optimization

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**Course objective.** This course provides a toolbox for solving dynamic optimization problems in economic models. In particular, we review the mathematical tools required for graduate courses in economics including control theory, and dynamic programming. We then study models in discrete and continuous time. Throughout the course, the optimization problems are illustrated using various examples. A particular focus will be on formulating and solving dynamic models in modern macroeconomics.

## Course outline.

### Part I: Basic mathematical tools

- (i) Topics in integration and differential equations (rules of transformation, differentiation, solution techniques)
- (ii) Qualitative theory (phase plane analysis, stability for nonlinear systems)
- (iii) Control theory (maximum principle, Euler equation, transversality condition)
- (iv) An example: Ramsey problem of optimal consumption
- (v) Dynamic programming (Bellman equation, envelope theorem, multiple variables)
- (vi) An example: Lucas' model of endogenous growth

### Part II: Stochastic models in discrete time

- (i) Topics in difference equations (basic concepts, solution techniques)
- (ii) Discrete-time optimization (dynamic programming, stochastic control problems)
- (iii) Overlapping-generations, Real-business-cycles

### Part III: Stochastic models in continuous time

- (i) Stochastic differential equations and rules for differentials (Itô's formula)
- (ii) An example: Merton's model of growth under uncertainty
- (iii) Stochastic dynamic control problems (Bellman equation)
- (iv) Optimal saving under uncertainty (Poisson uncertainty, Brownian motions)

**Reading list.** The main text books are Sydsæter et al. (2008, chap. 4-12, 290 pages) and Chang (2004, chap. 4, 50 pages). Reflecting the lecture notes is compulsory.

## References

Chang, Fwu-Ranq, *Stochastic optimization in continuous time*, Cambridge Univ. Press, 2004.

Sydsæter, Knut, Peter Hammond, Atle Seierstad, and Arne Strøm, *Further Mathematics for Economic Analysis*, Prentice Hall, 2008.