

Dynamic Optimization

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Assignment 1

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Exercise 1.1 Suppose $x^*(t)$ is a particular solution to $\dot{x} + a(t)x = b(t)$. Show that the general solution is the sum of the solution to the homogenous differential equation and the particular solution.

Exercise 1.2 Haavelmo's growth model leads to the description

$$\dot{k} = \gamma_1 b k^\alpha + \gamma_2 k, \quad (1)$$

where γ_1, γ_2, b and α are positive constants, $\alpha \neq 1$.

- (a) Solve for the capital stock $k(t)$ and document your solution technique.
- (b) Draw a phase diagram in the (k, \dot{k}) -plane and characterize the solution for different parameter values.

Exercise 1.3 Transform the following second-order differential equation in x into a system of two first-order differential equations in x and y .

$$\ddot{x} - \dot{x} + 2x = 2$$

Compute the equilibrium point (x^*, y^*) . Is it globally asymptotically stable?