Dynamic Optimization

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Summer course 2009

Assignment 2

July 01, 2009

Exercise 2.1 *Consider the control problem with capital adjustment costs,*

$$\max \int_0^\infty e^{-rt} \left(F(K_t) - \Phi(I_t)\right) dt \quad s.t. \quad \dot{K}_t = I_t - \delta K_t, \quad K(0) = K_0.$$

Assume F to be strictly concave, $F' > 0$ and $F'' < 0$. If the firm accumulates capital, it faces adjustment costs $\Phi(I_t) = vI_t + I_t^2/2$, where $v > 0$.

(a) Solve the firm's optimization problem. Explain the economic trade-off involved in this particular problem using the Euler equation.

(b) Illustrate the model dynamics in the (K, I)-plane. Draw the zero motion lines and arrows indicating the directions of motion.

Exercise 2.2 *Suppose a project has to be finished at T which pays $R(T)$. Let M_T be a quality level required to get the reward. Generating $M(t)$ is costly because it involves some effort $E(t) \geq 0$,*

$$\dot{M} = f(E(t)), \quad M(0) = 0.$$

Assume that future payments are discounted at the constant rate ρ.

(a) Write down the control problem and the Bellman equation for the objective to maximize the present discounted value of the project.

(b) Solve the problem assuming that more effort leads to higher output, that is $f' > 0$, but with decreasing returns to scale, $f'' < 0$. Explain in words which behavior characterizes the optimal effort over time.

(c) Assume the output function $f(E(t)) = E^\alpha(t)$ where $0 < \alpha < 1$. Solve explicitly for the optimal level of effort over time of the project, and interpret your results economically.