

Dynamic Optimization

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Summer course 2009

Assignment 3

July 02, 2009

Exercise 3.1 Consider the following infinite horizon control problem of a representative consumer choosing the optimal consumption path $\{c_t\}_{t=0}^{\infty}$,

$$\max E \sum_{t=0}^{\infty} \beta^t u(c_t, v_t), \quad \text{s.t.} \quad a_{t+1} = (1 + r_t)a_t + w_t - c_t, \quad a_0, v_0 \text{ given.}$$

$v_t = c_{t-1}^{\gamma}$, $\gamma \geq 0$ denotes a specific habit. Suppose $\beta > 0$, whereas w_t and r_t are exogenous and deterministic processes, respectively.

- (a) Write down the optimal control problem and the Bellman equation for the objective to maximize the present discounted life-time utility.
(Hint: you may consider the habit as an additional state variable)
- (b) Derive a rule for optimal consumption. Explain your results in words.
- (c) Assume that the individual lives in a deterministic world and suppose that instantaneous utility has the following isoelastic form,

$$u(c_t, v_t) = \frac{(c_t/v_t)^{1-\alpha}}{1-\alpha}, \quad \alpha > 0$$

Write down the rule of optimal consumption. Explain your results and give an economic interpretation to the utility function.

- (d) Specify parameters such that instantaneous utility is time-separable and write down the optimal rule of consumption.